Method

Stage 1: Mental multiplication using partitioning

Informal recording in Year 3 might be:

\[
\begin{array}{c}
43 \\
\downarrow \\
40 + 3 \\
\downarrow \\
240 + 18 = 258
\end{array}
\]

Also record mental multiplication using partitioning:

\[43 \times 6 = (40 + 3) \times 6 = (40 \times 6) + (3 \times 6) = 240 + 18 = 258\]

Mental methods for multiplying TU × U can be based on the distributive law of multiplication over addition. This allows the tens and ones to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the ones can be multiplied first but it is more common to start with the tens.

Note: These methods are based on the distributive law. Children should be introduced to the principle of this law (not its name) in Years 2 and 3, for example when they use their knowledge of the 2, 5 and 10 times-tables to work out multiples of 7:

\[
\begin{array}{c}
\bigstar \bigstar \bigstar \bigstar \bigstar \bigstar \bigstar \bigstar \bigstar \\
\bigstar \bigstar \bigstar \bigstar \bigstar \bigstar \\
\bigstar \bigstar \bigstar \\
\bigstar \\
\bigstar \\
\bigstar \\
\bigstar \\
\bigstar \\
\bigstar \\
\bigstar \bigstar \bigstar \bigstar \bigstar \bigstar \bigstar \bigstar \bigstar \\
\bigstar \bigstar \bigstar \bigstar \bigstar \bigstar \\
\bigstar \bigstar \bigstar \\
\bigstar \\
\bigstar \\
\bigstar \\
\bigstar \\
\bigstar \\
\bigstar \bigstar \bigstar \bigstar \bigstar \bigstar \bigstar \bigstar \bigstar \\
7 \times 3 = (5 + 2) \times 3 = (5 \times 3) + (2 \times 3) = 15 + 6 = 21
\end{array}
\]

Stage 2: The grid method

- As a staging post, an expanded method which uses a grid can be used. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps.
- It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.
Method

- The next step is to move the number being multiplied (38 in the example shown) to an extra row at the top. Presenting the grid this way helps children to set out the addition of the partial products 210 and 56.

<table>
<thead>
<tr>
<th></th>
<th>30 + 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>×</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>210</td>
</tr>
<tr>
<td></td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>266</td>
</tr>
</tbody>
</table>

- The grid method may be the main method used by children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.

Stage 3: Expanded short multiplication

- The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.

<table>
<thead>
<tr>
<th></th>
<th>30 + 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>×</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>210</td>
</tr>
<tr>
<td></td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>266</td>
</tr>
</tbody>
</table>

- Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3 × 7 should be stressed.

- Most children should be able to use this expanded method for TU × U by the end of Year 4.

Stage 4: Short multiplication

<table>
<thead>
<tr>
<th></th>
<th>38</th>
</tr>
</thead>
<tbody>
<tr>
<td>×</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>266</td>
</tr>
</tbody>
</table>

- The recording is reduced further, with carry digits recorded below the line.

- If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 3.

Stage 5: Two-digit by two-digit products

- Extend to TU × TU, asking children to estimate first.

<table>
<thead>
<tr>
<th></th>
<th>38</th>
</tr>
</thead>
<tbody>
<tr>
<td>×</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>266</td>
</tr>
</tbody>
</table>

- Start with the grid method. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product. 56 × 27 is approximately 60 × 30 = 1800.
Method

- As in the grid method for TU × U in stage 4, the first column can become an extra top row as a stepping stone to the method below.

\[
\begin{array}{c|ccc}
& 20 & 7 \\
\hline
50 & 1000 & 350 & 1350 \\
6 & 120 & 42 & 162 \\
\hline
& & & 1512 \\
\end{array}
\]

- Reduce the recording, showing the links to the grid method above.

\[
\begin{array}{c|ccc}
& 20 & 7 \\
\hline
50 & 1000 & 350 & 1350 \\
6 & 120 & 42 & 162 \\
\hline
& & & 1512 \\
\end{array}
\]

56 × 27 is approximately 60 × 30 = 1800.

- The carry digits in the partial products of 56 × 20 = 120 and 56 × 7 = 392 are usually carried mentally.
- The aim is for most children to use this long multiplication method for TU × TU by the end of Year 5.

- Reduce the recording further.

\[
\begin{array}{c|cc}
\times & 27 \\
\hline
1000 & 50 \times 20 = 1000 \\
120 & 6 \times 20 = 120 \\
350 & 50 \times 7 = 350 \\
42 & 6 \times 7 = 42 \\
\hline
1512 \\
\end{array}
\]

56 × 27 is approximately 60 × 30 = 1800.

Stage 6: Three-digit by two-digit products

- Extend to HTU × TU asking children to estimate first. Start with the grid method.
- It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.

\[
\begin{array}{c|ccc}
\times & 20 & 9 \\
\hline
200 & 4000 & 1800 & 5800 \\
80 & 1600 & 720 & 2320 \\
6 & 120 & 54 & 174 \\
\hline
& & & 8294 \\
\end{array}
\]

286 × 29 is approximately 300 × 30 = 9000.
**Method**

Reduce the recording, showing the links to the grid method above.

- This expanded method is cumbersome, with six multiplications and a lengthy addition of numbers with different numbers of digits to be carried out. There is plenty of incentive to move on to a more efficient method.

\[
\begin{array}{c}
286 \\
\times 29 \\
\hline
4000 \\
1600 \\
120 \\
1800 \\
720 \\
54 \\
\hline
8294 \\
1
\end{array}
\]

286 \times 29 is approximately 300 \times 30 = 9000.

- Children who are already secure with multiplication for TU \times U and TU \times TU should have little difficulty in using the same method for HTU \times TU.

\[
\begin{array}{c}
286 \\
\times 29 \\
\hline
5720 \\
2574 \\
\hline
8294 \\
1
\end{array}
\]

- Again, the carry digits in the partial products are usually carried mentally.